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Ion- and electron-temperature fluctuations excited by resistive instabilities in a self-consistently stationary inhomogeneous plasma

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Ion- and electron-temperature fluctuations in a resistive plasma are analyzed in the two-fluid slab approximation, allowing for the self-consistently stationary system parameters. In the weakly collisional region, ion-temperature fluctuations which propagate in the direction of electron diamagnetic drifts can be excited, whereas electron-temperature fluctuations are considerably smaller in comparison with ion-temperature fluctuations and are out of phase with them. The modes are more stable when the electron drift (which produces magnetic shear, satisfying Ampere's law) flows in the opposite direction to the mode propagations. The negative electron-temperature gradient is found to be stabilizing, while the ion-temperature gradient is irrelevant to this stability. The density response exhibits Boltzmann-like behavior. In the strongly collisional region, both ion- and electron-temperature fluctuations can be excited to the same order and propagate in the ion diamagnetic drift direction. The negative ion-temperature gradient is destabilizing, whereas the negative electron-temperature gradient is stabilizing. The density response exhibits non-Boltzmann-like behavior.

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Low-frequency microscopic density fluctuations have been observed in a wide variety of magnetic confinement systems, such as tokamaks, stellarators, and multipoles. Some distinct features of such fluctuations have been identified as follows [1]: (1) the frequencies are of the order of the drift wave frequency, (2) the wave numbers are of the large perpendicular and small parallel wave numbers, (3) ion and electron features propagate in the ion and electron diamagnetic drift directions, respectively [2], (4) reversal of the plasma current direction reverses the up-down asymmetry in the microscopic fluctuations [3], and (5) the density fluctuations exhibit non-Boltzmann-like behavior [4]. More recently, temperature fluctuations have been extensively measured in some magnetic confinement schemes [5–8].

Galeev, Oraevskii, and Sagdeev [9] discovered the ion-temperature gradient instability in a collisionless plasma for which the ion-temperature fluctuations are excited by the ion-temperature gradient. Later, Tsai, Perkins, and Stix [10] considered the electron-temperature fluctuations associated with the collisional drift instability in the uniform magnetic field, dropping the ion-temperature effects. A complete understanding of heat transport requires a knowledge of temperature fluctuations. Hence it is of particular interest to study electron-temperature fluctuations, systematically and simultaneously with ion-temperature fluctuations, because both electron and ion heat transports are observed to be anomalous [1,2]. In this article we analyze ion- and electron-temperature fluctuations in a resistive plasma which retains the electron- and ion-temperature inhomogeneities, magnetic shear, and plasma currents, distinguishing two regions: the weakly and the strongly collisional regions.

In the weakly collisional region, the plasma under consideration is supposed to be immersed in a sheared magnetic field in slab geometry. Most analyses of drift waves in a sheared magnetic field were concerned with localized modes, which are centered on the mode rational surfaces on which the parallel wave number is zero [11–13]. In this model, both magnetic shear and the parallel wave numbers of drift waves increase from zero to infinity, with increasing distance from the mode rational surface.

The stationary condition for a plasma embedded in a static magnetic field requires that total pressure (magnetic pressure plus plasma pressure) be constant. From this it follows that both the spatial variation in the values of the parallel wave numbers of fluctuations and the allowed values of magnetic shear are limited to the finite values [14,15], unlike the above model.

Earlier work indicates that resistive drift waves centered on the mode rational surface are stable [16]. So we are interested in the stability of alternative modes that are structurally different from such modes. In addition, the phase difference between the density response and the fluctuating electrostatic potential approaches zero at both small and large parallel wave numbers [17]. It is worth noting that this phase difference is the source of drift wave instability. As a result, we address the modes whose parallel wave numbers are small everywhere, and which are not localized near the mode rational surface, obtaining unstable electron drift waves in a sheared magnetic field [14,15].

In the weakly collisional region, ion-temperature fluctuations which propagate in the direction of electron diamagnetic drifts are shown to be excited, whereas electron-temperature fluctuations are considerably smaller in comparison with ion-temperature fluctuations and are out of phase with them. The growth rate is expressed as superposition (integral) of the local growth rate, resulting from the scattering (Cherenkov emission) of the lowest-order stable modes by small resistivity. The modes are more stable when the electron drift (which produces magnetic shear, satisfying Ampere's law) flows in the opposite direction to the mode propagations [14,15]. The negative electron-temperature gradient is found to be stabilizing, while the ion-temperature gradient is irrelevant to this stability. The density response exhibits Boltzmann-like behavior.

In the strongly collisional region, both ion- and electron-temperature fluctuations can be excited to the same order and propagate in the ion diamagnetic drift direction. The negative ion-temperature gradient is destabilizing, whereas the negative electron-temperature gradient is stabilizing. The density response exhibits non-Boltzmann-like behavior.

Finally, it is remarked that the ion-temperature-gradient-driven (reactive) instability [9] is excluded from our considerations.

We begin our discussion by considering a self-consistently stationary state of a low-β plasma immersed in a sheared magnetic field, which retains the ion- and electron-temperature inhomogeneities, and a plasma current self-consistent with magnetic shear. A two-fluid description of the plasma in a slab geometry is used, with the inhomogeneity in the x direction. The sheared magnetic field $\mathbf{B}(x)=B_y(x)\mathbf{y}+B_z(x)\mathbf{z}$ ($|B_y|\ll|B_z|$) satisfies Ampere's law:

$$\frac{d}{dx}B_y(x)=\beta N(x)[V_z(x)-V_{ez}(x)] ,$$
$$\frac{d}{dx}B_z(x)=-\beta N(x)[V_y(x)-V_{ey}(x)] . \tag{1}$$

Here B_y and B_z are normalized to $B(0)=[B_y^2(0)+B_z^2(0)]^{1/2}$; $N(x)$ is the normalized stationary density [$N(0)=1$], assumed to be the same for ions and electrons; β is the ratio of electron plasma pressure $N(0)T_e(0)$ to magnetic pressure at $x=0$. $\mathbf{V}$ is the stationary ion drift velocity in units of the sound velocity c_s ($\propto[T_e(0)]^{1/2}$), the subscript e refers to the electron species, and space is expressed in units of $c_s\Omega_i^{-1}$ (Ω_i is the ion Larmor frequency at the origin). Substituting these into the momentum equation in a stationary state yields

$$T(x)N(x)+\frac{1}{2\beta}B^2(x)=T(0)N(0)+\frac{1}{2\beta}B^2(0)$$
$$=T(0)+\frac{1}{2\beta} \tag{2}$$

and

$$V_y-V_{ey}=\frac{1}{NB_z}\left[\frac{d}{dx}(TN)+\frac{B_y}{\beta}\frac{dB_y}{dx}\right] , \tag{3}$$

where $T_i(x)$ is the ion temperature and $T\equiv T_i+T_e$. Equations (1) and (3) represent the stationary drifts in terms of the pressure inhomogeneity and magnetic shear. It should be emphasized that in a low-β plasma the magnetic field can vary only slightly over distance of the order of the plasma pressure scale length and the allowed values of magnetic shear B_y lie within the order of $\beta^{1/2}$ ($\ll 1$) when the variation in the normalized pressure $T(x)N(x)$ is of order unity.

The linearized equations governing the evolution of temperature fluctuations are taken to be the two-fluid equations derived by Braginskii [18]:

$$(N+n)\left[\frac{\partial}{\partial t}\mathbf{v}+(\mathbf{V}+\mathbf{v})\cdot\nabla\mathbf{v}\right]+\nabla[(T_i+t_i)(N+n)]+\nabla:\pi+(N+n)\left[\nabla\phi-\mathbf{v}\times\mathbf{B}+\nu(\mathbf{v}_p-\mathbf{v}_{pe})-c_1\frac{\partial}{\partial\mathbf{p}}t_e\right]=0 , \tag{4}$$

$$\nabla\phi-\frac{1}{N+n}\nabla[(T_e+t_e)(N+n)]-\mathbf{v}_e\times\mathbf{B}+\nu(\mathbf{v}_p-\mathbf{v}_{pe})-c_1\frac{\partial}{\partial\mathbf{p}}t_e=0 , \tag{5}$$

$$\frac{\partial}{\partial t}n+\nabla\cdot[(N+n)(\mathbf{V}+\mathbf{v})]=0 , \tag{6}$$

$$\frac{\partial}{\partial t}n+\nabla\cdot[(N+n)(\mathbf{V}_e+\mathbf{v}_e)]=0 ,$$

$$\tfrac{3}{2}(N+n)\left[\frac{\partial}{\partial t}+(\mathbf{V}+\mathbf{v})\cdot\nabla\right](T_i+t_i)+(N+n)(T_i+t_i)\nabla\cdot\mathbf{v}+\tfrac{5}{2}\nabla\cdot[(N+n)(T_i+t_i)\mathbf{p}\times\nabla(T_i+t_i)]=0 , \tag{7}$$

$$\tfrac{3}{2}(N+n)\left[\frac{\partial}{\partial t}+(\mathbf{V}_e+\mathbf{v}_e)\cdot\nabla\right](T_e+t_e)+(N+n)(t_e+t_e)\nabla\cdot\mathbf{v}_e$$
$$-\tfrac{5}{2}\nabla\cdot[(N+n)(T_e+t_e)\mathbf{p}\times\nabla(T_e+t_e)]-c_1NT_e\frac{\partial}{\partial p}(v_p-v_{pe})-\frac{c_2}{\nu}NT_e\frac{\partial^2}{\partial p^2}t_e=0 . \tag{8}$$

Here the lowercase symbols $\mathbf{v}_e$, t_e, t_i, and n are the fluctuating electron fluid velocity, electron- and ion-temperature fluctuations, and density fluctuations, respectively; the normalized collisionless stress tensor π is given in the form

$$\pi_{xx}=-\pi_{ll}=-(N+n)(T_i+t_i)\left[\frac{\partial v_x}{\partial l}+\frac{\partial v_l}{\partial x}\right]\Big/2 ,$$

$$\pi_{xl}=\pi_{lx}=(N+n)(T_i+t_i)\left[\frac{\partial v_x}{\partial x}-\frac{\partial v_l}{\partial l}\right]\Big/2 ,$$

where l indicates the component normal to both the magnetic field and the x axis, and $\partial/\partial l\equiv(B_z/B)\partial/\partial y-(B_y/B)\partial/\partial z$; ν denotes the electron-ion collision frequency; p indicates the component parallel to the magnetic field, and $\partial/\partial p\equiv(B_y/B)\partial/\partial y+(B_z/B)\partial/\partial z$; c_1

and c_2 are numerical coefficients of order unity; time is expressed in units of Ω_i^{-1}. For simplicity, ion-ion collisions (viscosity) are dropped from the theory.

We assume that the wave frequency ω is much smaller than the ion Larmor frequency Ω_i. This eliminates the fluid velocities $\mathbf{v}_e$ and $\mathbf{v}$ from the above equations, as follows. For $O(\omega/\Omega_i)$,

$$v_x' = -i\frac{k_l}{B}\left[t_i + \frac{T_i}{N}n + \phi\right],$$

$$v_l' = \frac{1}{B}\left[\frac{\partial\phi}{\partial x} + \frac{T_i}{N}\frac{dN}{dx}n + \frac{1}{N}\frac{\partial}{\partial x}(Nt_i) + \frac{T_i}{N}\frac{\partial n}{\partial x}\right].$$

For $O((\omega/\Omega_i)^2)$,

$$v_x'' = \frac{1}{B}\left[-i\omega v_l' + \frac{d}{dx}\left[\frac{NT_i}{2B}\right]\left[\frac{\partial}{\partial x}v_x' + ik_l v_l'\right] + \frac{NT_i}{2B}\Delta v_x'\right],$$

$$v_l'' = \frac{1}{B}\left[-i\omega v_x' + \frac{d}{dx}\left[\frac{NT_i}{2B}\right]\left[ik_l v_x' - \frac{\partial}{\partial x}v_l'\right] - \frac{NT_i}{2B}\Delta v_l'\right],$$

where $k_l \equiv -i\partial/\partial l$ and $\Delta \equiv \partial^2/\partial x^2 - k_l^2$. Substituting these into the ion continuity equation (6) yields

$$\left[\omega\left[1 - \frac{T_i}{B^2}\Delta\right] - T\frac{k_p^2}{\omega} - \frac{1}{N}\frac{d}{dx}\left[\frac{NT_i}{B}k_l\right] - (\mathbf{k}\cdot\mathbf{V})\right]\frac{n}{N} - \left[\frac{\omega_d}{T_e} + \frac{\omega}{B^2}\Delta\right]\phi - \left[\omega_b + \omega\frac{\Delta}{B^2} + \frac{k_p^2}{\omega}\right]t_i - \frac{k_p^2}{\omega}t_e = 0, \quad (9)$$

where $k_p \equiv -i\partial/\partial p$, $\omega_d \equiv -T_e d(Nk_l/B)/N\,dx$, $\omega_b \equiv -(d/dx)(k_l/B)$, and we have used $v_p = (k_p/\omega)(Tn/N + t_i + t_e)$. It is noteworthy that both $|\omega_b/\omega_d|$ and the spatial variation in the values of k_p^2 lie within the order of β [14,15]. In the same way, the electron continuity equation reduces to

$$i\omega_c\left[\frac{T_e}{N}n - \phi + (1+c_1)t_e\right] = \left[-\omega + T\frac{k_p^2}{\omega} + \frac{1}{N}\frac{d}{dx}\left[\frac{NT_e}{B}k_l\right] + (\mathbf{k}\cdot\mathbf{V}_e)\right]\frac{n}{N} + \omega_d\frac{\phi}{T_e} + \left[-\omega_b + \frac{k_p^2}{\omega}\right]t_e + \frac{k_p^2}{\omega}t_i, \quad (10)$$

with $\omega_c \equiv k_p^2/\nu$. On the other hand, the ion- and electron-temperature equations (7) and (8) become, respectively,

$$\left[\frac{3}{2}\omega - T_i\frac{k_p^2}{\omega}\right]t_i - \frac{3}{2}\omega_t\phi - T_i\frac{k_p^2}{\omega}\left[\frac{T}{N}n + t_e\right] = 0, \quad (11)$$

$$\left[\frac{3}{2}\omega - T_e\frac{k_p^2}{\omega}\right]t_e - \frac{3}{2}\omega_{te}\phi - T_e\frac{k_p^2}{\omega}\left[\frac{T}{N}n + t_i\right] + iT_e\omega_c\left[c_3 t_e + (1+c_1)\left[\frac{T_e}{N}n - \phi\right]\right] = 0, \quad (12)$$

where $\omega_t \equiv -(k_l/B)(dT_i/dx)$, $\omega_{te} \equiv -(k_l/B)(dT_e/dx)$, and $c_3 \equiv c_2 + (1+c_1)^2$. Equations (9)–(12), being a complete set of equations for four unknowns n, ϕ, t_i, and t_e, constitute the basis of our analyses.

We distinguish between weakly and strongly collisional regions. We shall first focus our attention on the weakly collisional region ($\omega_c \gg \omega$). When $\omega_c \to \infty$, Eqs. (10) and (12) turn out to be, respectively,

$$\phi - \frac{T_e}{N}n - (1+c_1)t_e = 0, \quad \phi - \frac{T_e}{N}n - \frac{c_3}{1+c_1}t_e = 0,$$

showing that electron-temperature fluctuations are negligibly small (isothermal) in this approximation and electrons obey a Boltzmann distribution. On taking into account a small dissipation, we have electron-temperature fluctuations to order ω_c^{-1},

$$t_e = \frac{i}{c_2\omega_c}\left[\frac{3}{2}(1+c_1)\omega_{te}\left[-\omega + \frac{1}{N}\frac{d}{dx}\left[\frac{NT_e}{B}k_l\right] + (\mathbf{k}\cdot\mathbf{V}_e)\right] + \frac{c_1(\omega_t + T\omega)}{\dfrac{\omega^2}{k_p^2} - \dfrac{2}{3}T_i}\right]\frac{n}{N}.$$

On the other hand, ion-temperature fluctuations in the first approximation can be written as

$$t_i = \left[\frac{3}{2}T_e\omega_t + T_i T\frac{k_p^2}{\omega}\right]\frac{n}{N}\Big/\left[\frac{3}{2}\omega - T\frac{k_p^2}{\omega}\right].$$

It is seen that ion-temperature fluctuations are significantly larger (adiabatic) in comparison with electron-temperature fluctuations, and out of phase with them. Now, upon inserting these into Eq. (9), we obtain the eigenmode equation as

$$\left\{\omega^3\left[-1 + \left[T + T_e\frac{\omega_t}{\omega}\right]\frac{\Delta}{B^2}\right] + \omega^2(\omega_d + T_i\omega_b) + \omega(T'k_p^2 - \omega_t\omega_b) + \frac{2}{3}T_i\left[\omega_d - T_e\omega_b - \frac{3T_e}{2T_i}\omega_t\right]k_p^2\right\}n = 0, \quad (13)$$

with $T' \equiv T + 2T_i/3$. Here we shall confine our analysis to fluctuations with $|\omega| \sim |\omega_d| \sim |\omega_{te}| \sim |\omega_t|$ and $|\Delta| \gg |k_p^2|$. If we set $\Delta/B^2 = -k_n^2$ in the above equation, to good approximation we have the local eigenfrequency

$$\omega = \omega_d(1 - Tk_n^2) + T'\frac{k_p^2}{\omega_d} + \left[T_i \left[1 - \frac{2k_p^2}{3\omega_d} \right] \omega_b + \left[k_n^2 - \frac{k_p^2 + \omega_b}{\omega_d} \right] \omega_t \right] .$$

The second term indicates that the electrons behave isothermally, while the ions respond adiabatically. The third term is related to magnetic shear and the ion-temperature gradient, and vanishes as $B_y \to 0$ and $dT_i/dx \to 0$.

According to the perturbation theory [14,15], the growth rate can be expressed as superposition (integral) of the local growth rate, which is taken as the scattering (Cherenkov emission) of the lowest-order stable mode of Eq. (13) by small resistivity

$$\gamma = \frac{\int dx \frac{\omega_d}{c_2\omega_c} n_0^2 \{ c_3[(\mathbf{k}\cdot\mathbf{V}_e) - \omega_0] - \frac{3}{2}(1+c_1)\omega_{te} \}}{\int dx\, n_0^2}$$

$$= \frac{\int dx \frac{\omega_d}{c_2\omega_c} n_0^2 \left\{ c_3 \left[\left[-\frac{k_y T_e}{B_z N}\frac{dN}{dx} \right] - \frac{(\mathbf{k}\cdot\mathbf{B})}{\beta N B_z}\frac{dB_y}{dx} - \omega_0 \right] - \frac{3}{2}(1+c_1)\omega_{te} \right\}}{\int dx\, n_0^2} ,$$

where n_0 and ω_0 are stable solutions of Eq. (13) and it is assumed that magnetic shear is related only to the electron drift. The negative electron-temperature gradient is found to be stabilizing, while the ion-temperature gradient is irrelevant to this stability. The first term in the curly brackets indicates that mode stability arises from a competition between the Doppler shifts resulting from both the electron diamagnetic drift and the electron drift related to magnetic shear, and the wave frequency, and we see that the modes are more stable when the electron drift flows in the opposite direction to the mode propagation [14,15].

We come now to examine the strongly collisional region ($\omega_c \ll \omega$). To make the physical contents clear, we shall here ignore magnetic shear. Subtracting Eq. (9) from Eq. (10) gives, to lowest order,

$$T_i\frac{n}{N} + \phi + t_i = 0 ,$$

from which we see that the fluctuating density exhibits non-Boltzmann-like behavior. The ion- and electron-temperature fluctuations can then be written as, respectively,

$$t_i = -T_i \left[1 - \frac{\omega}{D} \right] \frac{n}{N} ,$$

$$t_e = -T_e \left[1 - \left[1 + \frac{T_i}{T_e}\frac{\delta}{\omega} \right] \frac{\omega}{D} \right] \frac{n}{N} ,$$

where

$$D \equiv \omega - \frac{2}{3}T\frac{k_p^2}{\omega} + \frac{\omega_t}{T_e} - \frac{2}{3}T_i\delta\frac{k_p^2}{\omega^2}$$

and

$$\delta \equiv \frac{\omega_t}{T_i} - \frac{\omega_{te}}{T_e} .$$

We observe that electron-temperature fluctuations are comparable to ion-temperature fluctuations, provided that $T_i \sim T_e$. Substituting these into Eq. (10), we find the dispersion relation

$$\omega_0^2 + (\omega_{di} + \omega_t)\omega_0 - \frac{5}{3}\left[T + T_i\frac{\delta}{\omega_0} \right] k_p^2 = 0 ,$$

with $\omega_{di} \equiv T_i\omega_d/T_e$. We note here that the mode propagates in the ion diamagnetic direction ($\omega_{di} + \omega_t > 0$). The third term shows that both ion and electron fluctuations behave adiabatically. On taking into account small ω_c, we find the growth rate

$$\gamma = \frac{\omega_c T_i \omega_d}{2|\omega_0(\omega_0 - \omega_t)|} \left[\frac{T}{T_i T_e}\omega_0 + \frac{\omega_t}{T_i} - (1+c_1)\frac{\omega_{te}}{T_e} \right] .$$

This indicates that the negative ion-temperature gradient is destabilizing, whereas the negative electron-temperature gradient is stabilizing.

Finally, we state that, in the intermediate region, the two modes discussed here are expected to coexist.

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